The Physics of Bungee Jumping

Outcomes:
1. Analyze natural and technological systems to interpret and explain their structure. (116-7)
2. Describe and evaluate the design of technological solutions and the way they function, using energy principles. (116-6)
3. Analyze and describe examples where technological solutions were developed based on scientific understanding. (116-4)
4. Distinguish between problems that can be solved by the application of physics-related technologies and those that cannot. (118-8)
5. Analyze and describe examples where energy-related technologies were developed and improved over time. (115-5, 116-4)
6. Analyze the risks and benefits to society and the environment when applying scientific knowledge or introducing a particular technology (118-2)
7. Construct and test a prototype of a device and troubleshoot problems as they arise. (212-14)
8. Analyze quantitatively the relationships among mass, height, gravity, spring constant, gravitational potential energy and elastic potential energy. (326-1)
9. Solve problems using the law of conservation of energy, including changes in elastic potential energy.

Introduction

Would you plunge off a bridge attached only by a soft springy cord that could stretch three to four times its free length? If you understood the physics behind such a daring feat you just might! Bungee jumping involves attaching oneself to a long cord and jumping from extreme heights. It is related to a centuries old practice from the Pentecost Island in the Pacific Archipelago of Vanuatu. On this island, the men jump to show their courage and to offer thanks to the gods for a good harvest of yams. In 1979, members of the Oxford University Dangerous Sport Club jumped off a bridge near Bristol, England, apparently inspired by a film about “vine jumpers”. In the early 1990’s, the sport gained popularity in the United States and Canada. Today it is still dubbed the “ultimate adrenaline rush” (Menz, 1993).

Equipment

The old adage of “less is more” certainly applies to bungee jumping. The only equipment required is a springy cord and a harness. However it is very important that the equipment used be strong and secure. The harnesses are similar to those used in mountain climbing, including the carabiner which is the main link between the cord and the harness. The cord itself is soft and springy and is secured tightly to the jumper’s body. Jumpers today are typically aided by double hookups. If an ankle jump is chosen, the body harness is used as a backup. If the body harness is chosen, a chest/shoulder harness becomes the backup.

Though there have been some accidents related to bungee jumping (three deaths in France in 1989), they can be traced to human error in attachment, total height of jump available, or a mismatch between the cord and jumper. Minor injuries like skin burn or being hit by the cord happen when
jumps do not follow instructions. Skin burn for example is caused by gripping the cord. Understanding and adhering to some basic physics principles would prevent such problems.

**Theory**

**Energy Distribution**

The main physics concepts involved in bungee jumping are the gravitational potential energy of the jumper and the elastic potential energy of the stretched cord. Initially the jumper is attached to the cord which is attached to a supporting structure on the same level as the jumper’s center of mass. Standing on the platform, the jumper possesses gravitational potential energy given by,

\[ E_p = mgh \]

where \( h \) is the height from the top to the bottom extremity of the jump. At the beginning of the jump (before the cord reaches maximum length) the jumper experiences free fall. In free fall the only force acting on the jumper (neglecting air friction) is the force of gravity which causes the person to accelerate downward at 9.8 m/s\(^2\). Free fall is a funny sensation in that the jumper experiences no outside forces and thus their internal organs are not pushing on each other. The free fall typically lasts between one and two seconds. During this time the bungee cord is not yet stretching and some of the original gravitational potential energy is transferred into kinetic energy \( \frac{1}{2}mv^2 \). The distribution of energy at a certain height “d” is then given by,

\[ E_{total} = mgd + \frac{1}{2}mv^2 \]

When the cord reaches its full length it begins to stretch and applies an upward force that begins to slow the jumper. At this point some of the jumper’s energy is stored in the bungee cord \( \frac{1}{2}kx^2 \) and the total energy is given by,

\[ E_{total} = mgd + \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \]

When the jumper reaches the bottom extremity of the jump the velocity of the jumper, and therefore the kinetic energy, is zero. At that point the gravitational potential energy possessed at the top has been totally converted into the elastic potential of the cord. Since energy is conserved in the jump, the gravitational potential energy of the jumper must equal the elastic potential energy of the cord.

\[ E_{top} = E_{bottom} \]

\[ mgh = \frac{1}{2}kx^2 \]

The elastic potential energy refers to the energy stored in the cord by virtue of stretching it. The jumper will realize that there is stored energy in the cord when it rebounds to its equilibrium shape. The restoring force of the cord is used to decelerate and eventually stop the jumper.

The figure below (Nowikow & Heimbecker, 2001) shows how the different types of energy change during the jump. Note that as the gravitational potential energy decreases during the fall, the kinetic energy increases. At the bottom extremity of the fall as the cord tightens, the loss in gravitational potential energy is matched by a corresponding increase in the elastic potential energy of the bungee cord. At any point in the fall, the sum of the kinetic and elastic potential energies is equal to the gravitational potential energy lost during the fall.

**Hooke’s Law and Elastic Potential Energy**

The work done to stop the jumper is related to the stiffness of the bungee cord. The cord acts like a spring that obeys Hooke’s Law. Hooke’s Law is given by,

\[ F = kx \]

where \( F \) is the restoring force, \( k \) is the spring constant.
constant and \( x \) is the stretch of the cord. The elastic potential energy possessed by the cord at the bottom of the fall is given by,

\[
E_p = \frac{1}{2} kx^2
\]

Thus we can write that,

Potential energy at the top relative to the bottom of the fall. = Elastic potential energy of cord at the bottom extremity of the fall.

or mathematically,

\[
mg(h) = \frac{1}{2} kx^2
\]

\[
mg(L + x) = \frac{1}{2} kx^2
\]

where \( h = (L + x) \), \( L \) is the length of the bungee cord and \( x \) is the stretch of the bungee cord.

This relationship allows the correct matching of cord with person or of jump height with person. If for example a given jump height \((L + x)\) is to be matched with a given person of mass \( m \), we can determine what stiffness \((k)\) of cord should be used for that jump.

\[
mg(L = x) = \frac{1}{2} kx^2
\]

\[
kx^2 = 2mg(L + x)
\]

\[
k = \frac{2mg(L + x)}{x^2}
\]

If however a given cord of length \( L \) and stiffness \( k \) is to be matched with a person of mass \( m \), then the amount of stretch can be determined as follows,

\[
mg(L + x) = \frac{1}{2} kx^2
\]

\[
mgL + mgx = \frac{kx^2}{2}
\]

\[
2mgL + 2mgx = kx^2
\]

\[
kx^2 = 2mgx - 2mgL = 0
\]

\[
x = \frac{2mg \pm \sqrt{(2mg)^2 + 4k(2mgL)}}{2k}
\]

\[
x = \frac{2mg \pm \sqrt{4m^2g^2 + 8kmgL}}{2k}
\]

In most cases the latter method is the way the match would be made so that the total fall \((L + x)\) will fit the jumping facility.

Hooke’s Law can also be applied to determine the maximum force experienced by a jumper. If for example a 68 kg person is to jump using a 9.0 m cord which will stretch 18 m, we get the following.

\[
k = \frac{2mg(L + x)}{x^2}
\]

\[
k = \frac{2(68kg)(9.8\text{ m/s}^2)(9.0m + 27m)}{(27m)^2}
\]

\[
k = 111 \frac{N}{m}
\]

Therefore,

\[
F = kx
\]

\[
= (111 \frac{N}{m})(18m)
\]

\[
= 1998N
\]

Thus the force is about three times the person’s weight. A cord with more stretch would give a “softer” ride. If for example the stretch of the 9.0 m cord were 27 m,

\[
k = \frac{2mg(L + x)}{x^2}
\]

\[
k = \frac{2(68kg)(9.8\text{ m/s}^2)(9.0m + 27m)}{(27m)^2}
\]

\[
k = 66 \frac{N}{m^2}
\]

and

\[
F = kx
\]

\[
= (66 \frac{N}{m^2})(27m)
\]

\[
= 1782N
\]
This exerts a lesser force on the jumper for a more comfortable jump. In reality of course, one must consider that given facilities will have a limited number of cords of differing length and stiffness. Also, bungee cords have been found to demonstrate variable stiffness over their range of use (i.e. $k$ does not remain constant).

Oftentimes it is a matter of choosing the best fit available for the jump. It seems reasonable however to match heavier people with stiffer cords and lighter people with softer cords. The following table (Menz, 1993) illustrates this idea.

<table>
<thead>
<tr>
<th>Jumper Weight (N)</th>
<th>Cord</th>
<th>Stretch (m)</th>
<th>$F_{\text{max}}$ (N)</th>
<th>g’s</th>
<th>Jump Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1112</td>
<td>Stiff</td>
<td>17.9</td>
<td>3187</td>
<td>2.87</td>
<td>28.7</td>
</tr>
<tr>
<td>800</td>
<td>Medium</td>
<td>16.7</td>
<td>2311</td>
<td>2.99</td>
<td>27.5</td>
</tr>
<tr>
<td>490</td>
<td>Soft</td>
<td>13.8</td>
<td>1478</td>
<td>3.02</td>
<td>24.6</td>
</tr>
</tbody>
</table>

Menz (1993) recommends that a proper match of cord and jumper should produce maximum accelerations of the order of 3 g’s (where $g=9.8$ m/s$^2$).

**Conclusion**

Bungee jumping then deals with the conversion of gravitational potential energy into the elastic potential energy of a stretched cord. It is an extreme sport that requires courage, daring and a knowledge of physics – at least by the people organizing the jump.

**Questions**

1. A person of mass 65 kg is to bungee jump from a platform that is 18.5 m above the ground. If the bungee cord used has a stiffness of 204 N/m and a length of 9.5 m, is it safe for the person to jump?

2. A 75 kg person is to bungee jump with a cord of length 8.0 m that will stretch 10.0 m. What force will be exerted on the person?

3. Describe the energy conversions that take place as a person bungee jumps.

4. Research: What were bungee cords originally designed for?

**References**


Activity 1: Bungee Egg

**Purpose**: To drop an egg attached to a bungee cord and have it come as close to the ground as possible without breaking.

**Materials**:  
- ten rubberbands  
- meterstick  
- mass balance  
- sandwich baggie (to minimize mess)  
- masses  
- egg (or eggs)

**Procedure**:  
1. The procedure for this activity will be student designed. Their goal is to use energy calculations to determine the height of drop that will allow the egg to land safely. Students should be aware that the design of bungee jumps involves calculating the point at which the gravitational potential energy lost during the fall will equal the elastic potential energy gained by the elastic cord (or rubber bands). The following diagram may be useful in helping them visualize what they have to do. (Hint: Don’t forget to take into account the height of the “egg in bag” attachment).

Where $h$ is the total drop height, $l$ is a fixed string (optional), and $x$ is the amount of stretch.

![Diagram showing energy calculation](attachment:diagram.png)

$E_p = mg \cdot h$

$E_{ek} = \frac{1}{2} kx^2$

Raw eggs are dropped in a harness made from a sandwich baggie. Evaluate students on how close they come to the floor without breaking the egg.
Activity 1: Bungee Egg (continued)

Notes to Teacher:

1. The bungee cord can be constructed from rubber bands or a bungee cord if one is available. The piece of string of length l, may or may not be used. The rubber bands (or cord) could be attached directly to an adjustable platform (eg. ring stand). If the string is used we will ignore its stretch in the calculations.

2. Students will first determine the spring constant k for their bungee cord by hanging masses from the cord and measuring the amount of stretch. The slope of a graph of force versus stretch will then give the value of k. This is Hooke's Law.

3. Energy conservation principles can then be used to figure out the stretch of the cord at the bottom extremity of its fall:

\[ mgh = \frac{1}{2} kx^2 \]

\[ mg(L + x) = \frac{1}{2} kx^2 \]

where L is the length of the unstretched bungee cord and x is the amount of stretch.

4. Use the amount of stretch to determine what height the egg can be safely dropped from (by either adjusting the platform or adding a string to adjust the length).

5. Students should present calculations supporting their proposed drop height before any actual testing takes place.
Activities

Activity 2: Bungee Egg Drop 2

Refer to the following website (or see below) for another type of bungee egg drop experiment using a graphical analysis: http://www.physics.ucok.edu/~chughes/~plrc/Labs/BungeeEgg2.

Introduction:

You may have seen the bungee egg apparatus in a previous experiment: Bungee Egg Drop 1. The apparatus is constructed from a minimum 2 meter length of unstretchable string or cord, a minimum of 6 standard size rubber bands, one grade AA large egg, one small safety pin, and one “half size” ziplock bag. Briefly, the egg is placed in the plastic bag which is then attached to a string of rubber bands tied together. This is, in turn, attached to the unstretchable length of cord. Detailed instructions for constructing this object are presented below.

The egg is dropped by attaching the unstretchable cord to a support arm a chosen height above the floor. During the first part of the jump, only gravity acts on the egg causing it to fall faster and faster. When the egg has fallen a distance greater than the unstretched cord’s length, the cord pulls upward causing the egg to come to a stop somewhere above the ground (hopefully). This maximum height through which the egg is dropped can be adjusted by changing the length of the cord.

Constructing the Bungee Apparatus

The egg holder is constructed by taking a normal ziplock bag and cutting it lengthwise down the middle with a pair of sharp scissors:

One of the resulting half size bags will be used as the egg holder (the other one should be kept as a spare). To keep the egg from rolling out, it is necessary to tape the open side of the bag with transparent tape. This should be done carefully to make sure that the top still opens and closes with the ziplock.

Finally, a hole should be punched in the top of the bag with a hole punch. After placing the egg in the bag, a safety pin is attached to the bag through the hole.
Activities

Activity 2: Bungee Egg Drop 2 (continued)

Next the “bungee” part of the apparatus must be constructed. At least six standard rubber bands should be tied end-to-end to produce an elastic chain. One end of the rubber band chain is attached to the top of the safety pin. The other is tied to the piece of unstretchable cord.

The exact number of rubber bands, and the length of the string needed, must be chosen so that the bungee apparatus can cover the range of distances when the egg is dropped: from 100 cm to 500 cm.

The different maximum distances that the egg falls through are achieved by changing the length of the unstretchable part of the cord, that is, the length between the topmost rubber band and the point where the cord is tied to the support arm. As this distance gets bigger, the egg will fall through a larger distance.

It may take a little experimentation to find a workable combination of cord length and number of rubber bands that will effectively cover the range of values needed. One group of students may choose to have more rubber bands and shorter string lengths while another may opt for less rubber bands and a longer string length. The only restriction is that each group must have a minimum of six rubber bands and a minimum of 2 meters of string.

Springs

A rubber band doesn’t look much like a spring, but it really has behavior very similar to that of a spring.

Taking and Analyzing the Data

Ultimately you will have to use your measurements to make predictions. With this in mind, you might wonder about the best way to keep track of the data from the measurements.

You probably will want to start with a data table:

<table>
<thead>
<tr>
<th>d(cm)</th>
<th>L(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Activities

Activity 2: Bungee Egg Drop 2 (continued)

1. You should take enough data points so that the length L can be easily predicted to produce a given distance d in the range necessary for the contest. If the data points are too far apart, it becomes difficult to interpolate between the data.

2. The data table represents an insight into the real physics behind this apparatus. As such, it only gives a few pieces of information that point the way to explaining the relationship between L and d. The best way to use the data to explore this relationship is with a graph (as on the right). A graph of the data will show a collection of points on the graph. If enough points are found, a “curve” can be sketched through the data. While the line through the data is called a “curve,” the line can actually be curved or straight. Its shape depends on the underlying physical law that relates the quantities being plotted. The curve is the first clue about the nature of that physical law. It tells a theoretical physicist that “this is the information which your theory must match.”

For our purposes, the curve simply means an infinite number of data points, each representing how far an egg will drop for a given choice of string length. The curve should be a smooth line drawn through the data. Some data points might lie on either side of the final curve because of errors in measurement. This gives an indication of the uncertainty in your measurements and should be taken into account when selecting a string length for the contest.

The Contest

The graph of L vs d provides a visual description of the physics of your bungee apparatus. If you made careful measurements and took care to draw a neat curve through your data, you should be able to predict the length of string needed to cause the egg to drop exactly the distance d. Your bungee apparatus should be able to accurately predict the length of string necessary to make the egg fall, between 100 cm and 500 cm.

When you arrive for the contest, you will be shown the area where the egg will be dropped. You will then have 5 minutes to measure the height of the drop with a meter stick, determine the proper length of the string for your apparatus, and drop your egg. The winner will be the group of students with the egg that gets closest to the ground without cracking. Closeness to the ground will be judged by your instructor (whose opinion is final). Eggs must be raw and may not be cushioned in any way. The bag is merely to hold the egg. The eggs will be “tested for rawness” at the end of the competition (so don’t get too attached to your egg).